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LETTER TO THE EDITOR

An axisymmetric stationary solution of Einstein's equations calculated by computer

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Abstract. A computer programme is used to carry out further HKX rank-N transformations.

In a series of papers Kinnersley and his collaborators have explored the symmetry group of the space of stationary axisymmetric, vacuum gravitational fields. In particular Hoenselaers *et al* 1979a (actually in the sixth of the above mentioned series of papers and therefore referred to as VI) have given the so-called rank-N $(N = 0, 1, \ldots)$ transformations which generate non-trivial asymptotically flat solutions of Einstein's equations even when applied to flat space.

For the definition and derivation of the various expressions and for further references the reader is referred to VI.

The method of generating solutions uses the function

$$G(s,t) = -\frac{\mathrm{i}t}{2S(t)}\left(1+\frac{s+t-4stz}{sS(t)+tS(s)}\right)$$
$$S(t) = [(1-2tz)^2+(2t\rho)^2]^{1/2} \tag{1}$$

and its derivatives

$$G_{ij}(s,t) = \frac{s^i t^j}{i!\,j!}\,\partial_s^i \partial_t^j G(s,t) \tag{2}$$

for $i, j = 0, 1, \ldots, N$. From these functions one obtains a solution of the well known Ernst Equation as

$$\varepsilon = 1 + \mathrm{i}\sum_{p=0}^{N}\sum_{k=0}^{N-p}\sum_{l=0}^{N} G_{0,p}(0,u)\alpha_{p+k}M_{kl}^{-1}\,\partial_t G_{l0}(u,t)|_{t=0} \tag{3}$$

where α_i $(i = 0, \ldots, N)$ are arbitrary real constants and M_{kl}^{-1} is the inverse of the $(N+1)\times(N+1)$ matrix

$$M_{kl} = \delta_{kl} - \sum_{p=0}^{N-l} G_{kp}(u,u)\alpha_{p+l}. \tag{4}$$

This new solution contains $N+2$ parameters.

The result for $N = 0$ is just the extreme Kerr solution. The $N = 1$ solution has also been published (Hoenselaers *et al* 1979b) and analysed (Hoenselaers 1980). In these

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cases it turned out that the parameter u could be eliminated by the coordinate transformation $z \to z - 1/2u$.

According to (3) the derivation of a new solution with $N > 1$ does not require any particular intellectual effort. On the other hand, already the calculation of the $G_{ij}(u, u)$—not to mention the inversion of the matrix M_{kl}—very soon becomes so lengthy and involved a task that nobody has attempted to perform the calculations by hand.

I have developed the computer programme POLYNOM which can handle polynomials in a large number of variables analytically. It is capable of adding, multiplying, dividing and differentiating polynomials.

As one application of this programme, I have calculated the Ernst potential for $N=2$. At certain stages of the calculation the number of terms in some of the polynomials went up well above 100. The result, however, can be cast into the—compared with the number mentioned above—remarkably short form

$$\xi = \frac{1-\varepsilon}{1+\varepsilon} = \frac{\beta}{\alpha} \tag{5a}$$

$$\begin{aligned}\alpha = {} & r^9 + (aq - d^2) r^5 \sin^2\theta (1 + \cos^2\theta) + 4qdr^4 \sin^2\theta \cos\theta (1 + 2\cos^2\theta) \\ & - 2q^2 r^3 \sin^2\theta (2 - \cos^2\theta + 11\cos^4\theta) - \mathrm{i}[a\cos\theta r^8 - 2\,dr^7(\cos^2\theta - \sin^2\theta) \\ & + 4qr^6 \cos\theta (4\cos^2\theta - 3) - q^3 \sin^6\theta \cos\theta (3 + \cos^2\theta)]\end{aligned} \tag{5b}$$

$$\begin{aligned}\beta = {} & ar^8 - 2d\cos\theta r^7 + 2qr^6(3\cos^2\theta - 1) - q^3\sin^6\theta(1 + 3\cos^2\theta) \\ & + \mathrm{i}[2r^5 \sin^2\theta\cos\theta(aq - d^2) + 2qdr^4\sin^2\theta(7\cos^2\theta - 1) \\ & + 6q^2r^3\sin^2\theta\cos\theta(1 - 5\cos^2\theta)].\end{aligned} \tag{5c}$$

Here again it was possible to eliminate the parameter u entirely by the coordinate transformation given above and a renaming of the parameters

$$\alpha_0 = 4a \qquad \alpha_1 = 16u(d + 4uq) \qquad \alpha_2 = 64qu^2.$$

Further particulars of the calculation and a detailed description of POLYNOM will be published elsewhere.

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